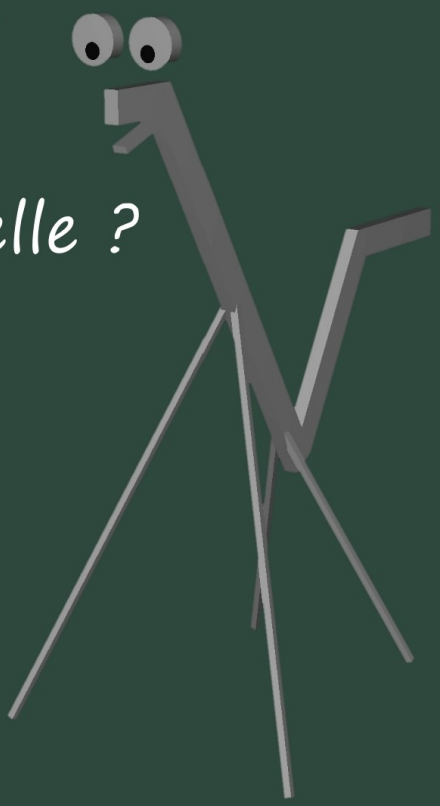
$$16^x + 20^x = 25^x$$

Une petite
équation
exponentielle ?



16^x + 20^x = 25^x

x = ?

RAISONNEMENT

16^x + 20^x = 25^x

16^x + 20^x - 25^x = 0

16^x[1 + (20/16)^x - (25/16)^x] = 0

16^x[1 + (5/4)^x - (25/16)^x] = 0

16^x = 0 => pas de solution pour x

1 + (5/4)^x - (25/16)^x = 0

1 + (5/4)^x - ((5/4)^2)^x = 0

rappel: (n^a)^b = (n^b)^a

1 + (5/4)^x - ((5/4)^x)^2 = 0

(-1)*(1 + (5/4)^x - ((5/4)^x)^2) = (-1)*0

((5/4)^x)^2 - (5/4)^x - 1 = 0

soit k = (5/4)^x alors ((5/4)^x)^2 - (5/4)^x - 1 = 0

... devient:

k^2 - k - 1 = 0

delta = (-1)^2 - 4*1*-1 = 1 + 4 = 5

racine #1 de k: k = (-(-1) + √5)/2*1 = (1 + √5)/2

racine #2 de k: k = (-(-1) - √5)/2*1 = (1 - √5)/2

k = (5/4)^x

racine #1 de x:

(5/4)^x = (1 + √5)/2

log((5/4)^x) = log((1 + √5)/2)

x*log(5/4) = log((1 + √5)/2)

x = [log((1 + √5)/2)]/log(5/4)

x = 2,15651

racine #2 de x:

(5/4)^x = (1 - √5)/2

log((5/4)^x) = log((1 - √5)/2)

log((1 - √5)/2) < 0 => pas de solution

RÉSULTAT(S)

x = [log((1 + √5)/2)]/log(5/4) = 2,15651